Gyakorló feladatok megoldásai - 7.

MA1122f – 2004/05

1. (a) $$\sum_{k=1}^{\infty} -2\frac{(-1)^k \sin(kx)}{k} = \begin{cases} x, & x \in (-\pi, \pi) \\ 0, & x = -\pi, \pi \end{cases}$$

(b) $$\frac{1}{3}\pi^2 + \sum_{k=1}^{\infty} 4\frac{(-1)^k \cos(kx)}{k^2} = x^2, \qquad x \in [-\pi, \pi]$$

(c) $$3 + \sum_{k=1}^{\infty} 36\frac{(-1)^k \cos(\frac{1}{3}k\pi x)}{k^2\pi^2} + \sum_{k=1}^{\infty} 6\frac{(-1)^k \sin(\frac{1}{3}k\pi x)}{k\pi} = \begin{cases} x^2 - x, & x \in (-3, 3) \\ 9, & x = -3, 3 \end{cases}$$

(d) $$1 + \sum_{k=1}^{\infty} -12\frac{(-1)^k \sin(\frac{1}{3}k\pi x)}{k\pi} = \begin{cases} 2x + 1 & x \in (-3, 3) \\ 1 & x = -3, 3 \end{cases}$$

(e) $$\frac{1}{6}\sin(6) + \sum_{k=1}^{\infty} -12\frac{\sin(6)\,(-1)^k \cos(\frac{1}{2}k\pi x)}{k^2\pi^2 - 36} = \begin{cases} \cos 3x, & x \in (-2, 2) \\ \cos 6, & x = -2, 2 \end{cases}$$

(f) $$-\frac{1}{8}e^{-4} + \frac{1}{8}e^4 + \sum_{k=1}^{\infty} 4\frac{(-1)^k \left(e^4 - e^{-4}\right)\cos(\frac{1}{2}k\pi x)}{k^2\pi^2 + 16}$$
$$+\sum_{k=1}^{\infty} \frac{k\pi\,(-1)^k \left(e^4 - e^{-4}\right)\sin(\frac{1}{2}k\pi x)}{k^2\pi^2 + 16} = \begin{cases} e^{-2x}, & x \in (-2, 2) \\ \frac{1}{2}(e^4 + e^{-4}), & x = -2, 2 \end{cases}$$

(g) $$\frac{1}{2} + \sum_{k=1}^{\infty} \frac{\left(-1 + (-1)^k\right)\sin(kx)}{\pi k} = \begin{cases} \frac{1}{2}, & x = -\pi, 0, \pi \\ 1, & x \in (-\pi, 0) \\ 0, & x \in (0, \pi) \end{cases}$$

(h) $$\frac{1}{4}\pi + \sum_{k=1}^{\infty} \frac{\left((-1)^k - 1\right)\cos(kx)}{\pi k^2} + \sum_{k=1}^{\infty} -\frac{(-1)^k \sin(kx)}{k} = \begin{cases} 0, & x \in (-\pi, 0] \\ x, & x \in (0, \pi) \\ \frac{\pi}{2}, & x = -\pi, \pi \end{cases}$$

(i) $$\frac{1}{4} + \sum_{k=1}^{\infty} \frac{\left((-1)^k - 1\right)\cos(k\pi x)}{k^2\pi^2} + \sum_{k=1}^{\infty} -3\frac{(-1)^k \sin(k\pi x)}{k\pi} = \begin{cases} x, & x \in (-1, 0] \\ 2x, & x \in (0, 1) \\ \frac{1}{2}, & x = -1, 1 \end{cases}$$

(j) $$\frac{1}{2} + \sum_{k=1}^{\infty} -2\frac{\sin(\frac{1}{2}k\pi)\cos(\frac{1}{2}k\pi x)}{k\pi} + \sum_{k=1}^{\infty} 4\frac{\left(-(-1)^k + \cos(\frac{1}{2}k\pi)\right)\sin(\frac{1}{2}k\pi x)}{k\pi}$$
$$= \begin{cases} -1, & x \in (-2, -1) \\ 0, & x \in (-1, 1) \\ 3, & x \in (1, 2) \\ 1, & x = -2, 2 \\ -\frac{1}{2}, & x = -1 \\ \frac{3}{2}, & x = 1 \end{cases}$$

2. (a) koszinuszos: $1, \qquad x \in [0, \pi]$

szinuszos: $$\sum_{k=1}^{\infty} -2\frac{\left((-1)^k - 1\right)\sin(kx)}{\pi k} = \begin{cases} 1, & x \in (0, \pi) \\ 0, & x = 0, \pi \end{cases}$$

(b) koszinuszos: $$\frac{1}{3} + \sum_{k=1}^{\infty} 4\frac{(-1)^k \cos(k\pi x)}{k^2\pi^2} = x^2, \qquad x \in [0, 1]$$

szinuszos: $$\sum_{k=1}^{\infty} -2\frac{\left(k^2\pi^2(-1)^k - 2(-1)^k + 2\right)\sin(k\pi x)}{k^3\pi^3} = \begin{cases} x^2, & x \in [0, 1) \\ 0, & x = 1 \end{cases}$$

(c) koszinuszos: $$\frac{2}{\pi} + \sum_{k=2}^{\infty} -2\frac{\left((-1)^k + 1\right)\cos(kx)}{\pi(k^2 - 1)} = \sin x, \qquad x \in [0, \pi]$$

szinuszos: $\sin x, \qquad x \in [0, \pi]$

(d) koszinuszos: $$\frac{3}{2} + \sum_{k=1}^{\infty} -2\frac{\sin(\frac{1}{2}k\pi)\cos(\frac{1}{2}k\pi x)}{k\pi} = \begin{cases} 1, & x \in [0, 1) \\ 2, & x \in (1, 2] \\ \frac{3}{2}, & x = 1 \end{cases}$$

szinuszos: $$\sum_{k=1}^{\infty} -2\frac{\left(2(-1)^k - \cos(\frac{1}{2}k\pi) - 1\right)\sin(\frac{1}{2}k\pi x)}{k\pi} = \begin{cases} 1, & x \in (0, 1) \\ 2, & x \in (1, 2) \\ \frac{3}{2}, & x = 1 \\ 0, & x = 0, 2 \end{cases}$$

3. (a) $$\int_0^{\infty} -2\frac{(-\sin(\lambda) + \lambda\cos(\lambda))\sin(\lambda x)}{\pi\lambda^2}d\lambda = \begin{cases} x, & x \in (-1, 1) \\ 0, & |x| > 1 \\ -\frac{1}{2}, & x = -1 \\ \frac{1}{2}, & x = 1 \end{cases}$$

(b) $$\int_0^{\infty} 8\frac{\sin(5\lambda)\cos(\lambda x)}{\pi\lambda}d\lambda = \begin{cases} 4, & x \in (-5, 5) \\ 0, & |x| > 5 \\ 2, & x = -5, 5 \end{cases}$$

(c) $$\int_0^{\infty} 2\frac{(\cos(\pi\lambda) + \pi\lambda\sin(\pi\lambda) - 1)\cos(\lambda x)}{\pi\lambda^2}d\lambda = \begin{cases} |x|, & x \in (-\pi, \pi) \\ 0, & |x| > \pi \\ \frac{\pi}{2}, & x = -\pi, \pi \end{cases}$$

(d) $$\int_0^{\infty} \frac{\left(e^5\cos(5\lambda) + e^5\lambda\sin(5\lambda) - e^{-5}\cos(5\lambda) + e^{-5}\lambda\sin(5\lambda)\right)\cos(\lambda x)}{\pi(1 + \lambda^2)}$$
$$+ \frac{\left(-e^5\lambda\cos(5\lambda) + e^5\sin(5\lambda) + e^{-5}\lambda\cos(5\lambda) + e^{-5}\sin(5\lambda)\right)\sin(\lambda x)}{\pi(1 + \lambda^2)}d\lambda$$
$$= \begin{cases} e^x, & x \in (-5, 5) \\ 0, & |x| > 5 \\ \frac{e^{-5}}{2}, & x = -5 \\ \frac{e^5}{2}, & x = 5 \end{cases}$$

(e) $$\int_0^\infty -\frac{(-1+\lambda\sin(\pi\lambda)-\cos(\pi\lambda))\cos(\lambda x)}{\pi(\lambda^2-1)}+\frac{(\lambda\cos(\pi\lambda)+\lambda-\sin(\pi\lambda))\sin(\lambda x)}{\pi(\lambda^2-1)}d\lambda$$
$$=\begin{cases}\sin x, & x\in[-\pi,0)\\ \frac{1}{2}, & x=0\\ \cos x, & x\in(0,\pi)\\ -\frac{1}{2}, & x=\pi\\ 0, & |x|>\pi\end{cases}$$

(f) $$\int_0^\infty -2\frac{(\cos(\pi\lambda)-1)\sin(\lambda x)}{\pi\lambda}d\lambda=\begin{cases}-\frac{1}{2}, & x=-\pi\\ -1, & x\in(-\pi,0)\\ 0, & x=0\\ 1, & x\in(0,\pi)\\ \frac{1}{2}, & x=\pi\\ 0, & |x|>\pi\end{cases}$$